Opening Doors for Students Who Struggle in Algebra

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Session Outline

Research basis for supporting students who struggle in mathematics

Hands-on exemplar tasks
Algebraic thinking

With a partner:
1. Solve the problems
2. Think about the mathematical understandings embedded in the problems
3. Consider how students might respond and why they would respond that way
Students’ Responses

Jon said, “$m – 1$ is always greater than $1 – m$.” Do you agree with Jon?

Total 632 responded

A. Agree, because $m$ is a positive number. 122 (19.3%)
B. Agree, because you cannot substitute a negative number for $m$. 109 (17.2%)
C. Disagree, because $1 – m$ is greater than $m – 1$ when $m$ is a negative integer. 254 (40.2%)
D. Disagree, because these expressions are equivalent. 145 (22.9%)
Students’ Responses

If \( d - 243 = 542 \), what does \( d - 245 \) equal?

**Total 476 responded**

- Scored correct: 540  →  214 (45%)
- Scored partially correct: 785  →  20 (4.2%)
- Incorrect: 544  →  44 (9.2%)
- Other incorrect responses: 198 (41.6%)
Students’ Responses

The equation below is true when $x = 6$:

\[ x^3 + x = 222 \]

What value of $x$ will make the following equation true?

\[ (2x)^3 + 2x = 222 \]

Total: 295

\[ x = 3 \ (72, \ 24.4\%) \]
Bart said, “$w + 3$ is less than $5 + w$.”

**Total 468 responded**

- Always true: 175 (37.4%)
- Sometimes true: 218 (46.6%)
- Never true: 69 (14.7%)

Explain your answer. **Total 466 responded**

Correct explanation: 23 (4.9%)
Students’ Responses

Dan challenged Amy to write an equation that has a solution of 3. Which of Amy’s equations has a solution of 3?

**Total 575 responded**

A. \(4 - x = 10 - 3x\)  119 (20.7%)

B. \(3 + x = -(x + 3)\)  133 (23.1%)

C. \(-2x - 2 = 8\)  95 (16.5%)

D. \(x + 2 = 3\)  221 (38.4%)
Types of Student Understandings

**Procedural** - Student can perform a computation or algorithm by following a series of prescribed steps

**Conceptual** - Student understands the basis of why a computation or algorithm works. They can apply it later without reteaching. Student can identify, describe, and explain a big idea related to a topic or a class of problems. They see connections between and among ideas.

**Problem solving** - Student can solve a problem when there is no specific solution pathway or algorithm
Important Ideas for Middle and High School

At your tables, think about the most important mathematical understandings that students should leave with from 8th grade and/or 12th grade.

Discuss why they are important.

“If I have 3 bones and Mr. Jones takes away 2, how many fingers will he have left?”
Number Ideas

- Number flexibility
- Rational number concepts and operations
- Working with generalized quantities
- Reasoning abstractly
Concept of Variable

- Roles of a variable
- Focus on generalized quantities
Relational Thinking
Relational Thinking

\[
\begin{align*}
471 - 382 &= 474 - 385 \\
674 - 389 &= 664 - 379 \\
583 - 529 &= 83 - 29 \\
37 \times 54 &= 38 \times 53 \\
5 \times 84 &= 10 \times 42 \\
64 \div 14 &= 32 \div 28 \\
42 \div 16 &= 84 \div 32
\end{align*}
\]
Relational Thinking

\[
73 + 56 = 71 + d
\]

\[
67 - 49 = c - 46
\]

\[
234 + 578 = 234 + 576 + d
\]

\[
94 + 87 - 38 = 94 + 85 - 39 + f
\]

\[
92 - 57 = 94 - 56 + g
\]

\[
68 + 58 = 57 + 69 - b
\]

\[
56 - 23 = 59 - 25 - s
\]
Writing Expressions

- Enter the first three digits of your phone number.
- Multiply by 80.
- Add 1.
- Multiply by 250.
- Add the last four digits of your phone number.
- Repeat the above step.
- Subtract 250.
- Divide by 2.

Describe the number you have.
How did the problem work?
Flexible Thinking

Which of the following steps can you reverse without changing the result? Why?

1) Think of a number.
2) Subtract 7.
3) Add 3 more than the original number.
4) Add 4.
5) Multiply by 3.
6) Divide by 6.
Characteristics of Effective Lessons

• Introduce every topic with problem solving

• Every lesson includes five forms of communication
  o Reading
  o Speaking
  o Critical listening
  o Writing
  o Multiple representations
Characteristics of Effective Lessons

• Topics are connected
• Students have 8–15 days to move a concept to a skill
• Challenging problems for all students (accessible)
Bridging the Gap

Changing the way tasks are posed
Creating high expectations and accountability
Questioning techniques

• Factual questions comprise the majority of questions asked in a mathematics class
  • More than 145 questions in 48-minute class period
  • Less than 2 seconds for response

Dougherty & Foegen, 2010
Moving from Rote to Understanding

Typical skill task:

Simplify: $4(3 + 5y)$
Find an expression that can be simplified to \(12 + 20y\).
Questioning: Reversibility

- Promote the ability to think in different ways
- Give the answer, students create the problem
Questioning: Generalization

● Find an algebraic expression with 4 terms that can be simplified to one with 2 terms. What do you notice about the terms?

● Find an algebraic expression with 3 terms that cannot be simplified. What do you notice about the terms?
Questioning: Flexibility

Write an expression equivalent to $3b - 5$.

Write another expression equivalent to $3b - 5$. 
Questioning: Flexibility

Simplify:
1. $5(4x - 3)$
2. $5(4a - 3)$
3. $5(4y + 3)$
4. $5(3 + 4y)$
What are some of the characteristics of games your students find engaging and are meaningful mathematically?
Gamification: Criteria

- Mathematics complexity
  - Focuses on mathematical thinking
  - Incorporates mathematical representations and connections to concepts
  - Allows for multiple strategies to play
  - Does not incorporate speed
Gamification: Criteria

- **Mathematics complexity**
- **Engagement and Game Features**
  - Mathematics learning incorporated into game mechanics (playing the game is doing math)
  - Stimulating and appealing
  - Engages students

Adapted from Holly Pope, Digital Math Game Evaluation Framework, 2016
Gamification: Criteria

- Mathematics complexity
- Engagement and Game Features
- Game theme
  - No gender advantages or stereotypes
  - No fixed mindset messages
  - No negative aspects about culture, environment, or the world

Adapted from Holly Pope, Digital Math Game Evaluation Framework, 2016
CLOSE enCOUNTER

http://programs.crdg.hawaii.edu/algebra_revision/closeEncounter/level_1/story_html5.html
How High: Exploration

Consider the task.

What mathematical ideas would be promoted by using the task?
For more info---

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